# Letters

## Comments on "An Innovative Fast Powerful Method for Tackling Electromagnetic Eigenvalue Problems for Multistrip Transmission Lines"

Michal Mrozowski

In the above paper,[1] Casanueva and García applied a technique developed in [1] to calculate dispersion characteristics of multistrip transmission lines. Apart from the waveguiding structure used in numerical tests and the method applied to calculate the basis (both being not essential for the method introduced in [1]), the only departure from [1] is the technique for solving the resulting eigenvalue problem. Casanueva and García treat the problem as a system of homogeneous equations and advocate solving it by using singular value decomposition (SVD) combined with the search of minimum on the complex $\beta$ plane. Unfortunately, this in fact is a much worse and less straightforward way than that which was actually used to solve eigenvalue problems derived in [1] and [3]. While I certainly appreciate and welcome the fact that Casanueva and García recognized a large potential of the approach outlined in [1], as well as its simplicity and innovative character, it is felt that an apparent misunderstanding as to the numerical treatment of the resulting eigenvalue problem has to be clarified and I take this opportunity to spell out some details.

Let us recall a few basic facts from the guided wave theory [2]. Using $\omega$, $\beta$, and $u$ to denote the angular frequency, propagation factor, and field, respectively, the wave-propagation problem in a lossless waveguide can be expressed in a form of an operator pencil

$$\omega^2 \mathbf{X}_{\omega^2} u + \omega\beta \mathbf{X}_{\omega\beta} u + \omega \mathbf{X}_{\omega} u + \mathbf{X}_0 u = 0 \tag{1}$$

where boldface font is used to represent operators derived from Maxwell's equations. For bidirectional media, the pencil reduces to

$$\mathbf{L}u + \omega^2 u - \beta^2 \mathbf{S} u = 0. \tag{2}$$

Note that operators neither depend on $\beta$, nor $\omega$. This is why the only one form of the matrix equation that can result when the method of moments is applied to the above equations (provided the basis and testing functions do not depend on $\beta$ and $\omega$) is

$$\left(\underline{\underline{A}}_1 + \omega^2 \underline{\underline{A}}_2 - \beta^2 \underline{\underline{A}}_3\right)\underline{a} = 0. \tag{3}$$

With the method presented in [1], one gets a problem in the form (see [1] or [3] for details and the definition of matrices involved)

$$\left[\underline{\underline{G}}\left(\omega^2 \underline{\underline{I}} - \underline{\underline{\Omega}}^2\right) + \underline{\underline{S}}\,\underline{\underline{Z}}^2\right]\underline{a} - \beta^2 \underline{\underline{S}}\,\underline{a} = 0 \tag{4}$$

so $\underline{\underline{A}}_1 = \underline{\underline{S}}\,\underline{\underline{Z}}^2 - \underline{\underline{G}}\,\underline{\underline{\Omega}}^2$, $\underline{\underline{A}}_2 = \underline{\underline{G}}$ and $\underline{\underline{A}}_3 = \underline{\underline{S}}$.

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Obviously one can disregard the fact that the component matrices do not depend on $\beta^2$ and $\omega^2$ and treat (3) as a system of homogeneous equations with a coefficient matrix depending on $\omega^2$ and $\beta^2$, i.e.,

$$\underline{\underline{A}}_4\left(\beta^2, \omega^2\right)\underline{a} = 0. \tag{5}$$

Written like that, it appears that the problem is soluble by finding roots of a nonlinear equation resulting from the condition $\det \underline{\underline{A}}_4(\beta^2, \omega^2) = 0$. It has to be stressed that, although theoretically possible, this would be the least efficient and exotic way of treating matrix problems of the form of (3). Equally exotic is the approach to solving (3) suggested by Casanueva and García, who advocate searching for the location of minimal value of the smallest singular value of the matrix on the complex $\beta$-plane. While this may indeed bring about some advantages when applied to the spectral-domain approach (SDA) (this is the method Casanueva and García use for calculating the basis), where the operators (Green's functions) depend nonlinearly on $\beta^2$ and $\omega^2$, it should never be considered for solving (3). Equation (3) simply constitutes a generalized matrix eigenvalue problem and dispersion characteristics $\beta^2(\omega)$ and expansion coefficients for finding modal fields are determined in a straightforward manner by finding, for successive values of $\omega$, eigenvalues and eigenvectors of the generalized matrix eigenvalue problem (4) expressed as

$$\underline{\underline{A}}_5(\omega)\underline{a} = \beta^2 \underline{\underline{S}}\,\underline{a}. \tag{6}$$

For this purpose, one of the standard numerical techniques, e.g., the $QZ$ algorithm [4], can be used. In the $QZ$ algorithm, matrices in the pencil $(\underline{\underline{A}}_5, \underline{\underline{S}})$ are first reduced to the upper Hessenberg and upper triangular matrices via a series of orthogonal transformations. Next, the generalized Schur decomposition is found by iteratively reducing the subdiagonal entries of the upper Hessenberg matrix. Eigenvalues are then calculated as the ratio of the elements situated on the diagonals of the reduced matrix pencil. The eigenvector is computed via inverse iteration. The $QZ$ algorithm is implemented in a variety of libraries, as well as in commercial- and public-domain mathematical software. For matrices of the size $N \times N$, the numerical cost of the eigenvalue solver is of the order $O(N^3)$, and no zero finding or minima searching is involved.

As matrices in pencil (6) are indefinite [2], the $QZ$ algorithm may result in pairs of complex conjugate eigenvalues and, thus, it also directly finds complex modes. To sum up, to compute dispersion characteristics presented in [1] and [3], no homogeneous system of equations has ever been solved. The propagation constants of propagating, evanescent, and complex modes were found by applying either the $QZ$ or $QR$ algorithm to various matrix eigenvalue problems that can be derived form (4) by using a suitably selected basis (see [3] for a discussion of specific bases).

As for the application of the SVD for a fast solution of guided wave-propagation problems, the only sensible application of this technique I see in the context of the method developed in [1], is the generation of orthogonal bases. If the fields selected for expansion are taken for the same mode, but for different frequency points, then the expansion functions show some linear dependency. SVD may then be used to find a minimal set of orthogonal vectors spanning the same space as those partly linearly dependent modal fields. This set may then be used as basis functions in place of actual fields.

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## Authors' Reply

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In response to Mrozowski's comments on the above paper,[1] we agree with the general outline and accept that the numerical technique suggested by Mrozowski is more orthodox than the one we used in the above paper given the current state of mathematical research. We recognized the innovative nature of the approach in [1]; however, the same approach was not developed more fully until [2], which, as Mrozowski states, was published four years later. The above paper was also published in 2002, but was actually commenced in 1998 and first submitted at the start of 1999, long before the publication of [2]. Furthermore, far from seeking to discover the most convenient mathematical procedure possible, we tested the algorithm on various different planar structures in order to prove the large potential of the approach in the most diverse situations, e.g., in the analysis of microstrip, suspended microstrip, and finline, all with differing dielectric constants. The general aim, therefore, was to see whether the simplicity of the algorithm was also valid for a wider and more complex range of solutions, which would thus prove that the new approach could be an efficient procedure when the most convenient expansion functions are used.

After reviewing Mrozowski's comments, we have gone back to our original calculations and reworked them in accordance with Mrozowski's suggestions. Some of the results of these new calculations are shown here and they prove that the original results presented in the above paper are in keeping with the new results obtained via the approximation formulated by Mrozowski and the application of the $QZ$ algorithm, as shown in Fig. 1.

The latter mathematical procedure is undoubtedly far more appropriate. Nevertheless, the fact that the algorithm is also valid with the mathematical procedure used in the above paper clearly shows that it is the basic functions used in the approach that give it its real efficiency, rather than the mathematical procedure applied.

As far as the new results shown here are concerned, the following should be noted. Fig. 2 and its inset present $\beta^2$ as a function of frequency calculated as approximate (shown by a dashed curve) and as accurate (shown as a continuous curve). In Fig. 2, different modes have been used at the same frequency. In Fig. 3, on the other hand, a TEM mode and modes near the cutoff frequency have been applied. We can observe that the approximate results in Fig. 3 are closer to the accurate results than in Fig. 2, which is due to the fact that, for microstrip structures, in general, the use of a TEM mode and modes near the cutoff frequency is more efficient. All the above is in complete agreement with the theory and data presented in the above paper.

Moreover, we would like to conclude by stressing that Mrozowski's proposed algorithms are valid for planar transmission lines when both the spectral-domain approach and the singular-value-decomposition technique have been implemented to obtain an accurate set of basic functions.

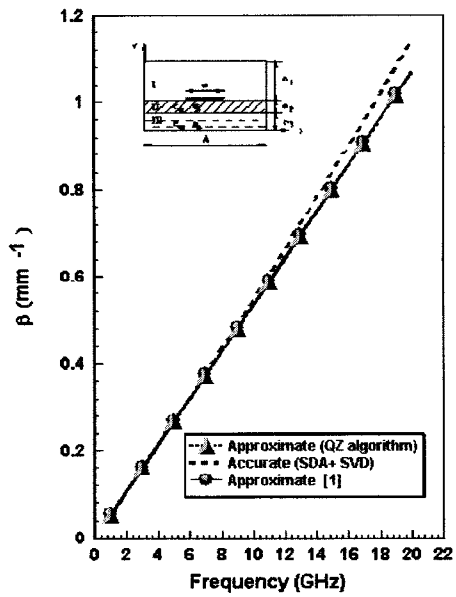


Fig. 1. Accurate and approximate data of propagation constant $\beta$ versus frequency in a shield microstrip line. Parameters: $h_1 = 3$ mm, $h_2 = h_3 = 0.3175$ mm, $w = 0.56$ mm, $A = 5$ mm, $\varepsilon_{r2} = \varepsilon_{r3} = 10$, $\sigma_2 = \sigma_3 = 0$. Basic functions: ($\beta_1 = 0.532991729\text{E-}01, 0.0$, Freq$_1$ = 1.00 GHz), ($\beta_2 = 0.305018254\text{E-}001, -3.231$, Freq$_2$ = 17.0 GHz).

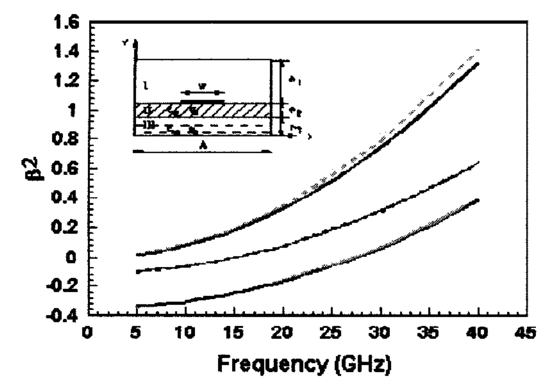


Fig. 2. Comparison of $\beta^2$ of the first three modes in a shield microstrip line between accurate and approximate data. Parameters: $h_1 = 5.751$ mm, $h_2 = h_3 = 0.3175$ mm, $w = 0.953$ mm, $A = 9.52$ mm, $\varepsilon_{r2} = \varepsilon_{r3} = 10$, $\sigma_2 = \sigma_3 = 0$. Basic functions: ($\beta_1 = 2.8872\text{E-}001, 0.0$, Freq$_1$ = 10.0 GHz), ($\beta_2 = 0.0, 2.4841\text{E-}001$, Freq$_2$ = 10.0 GHz), ($\beta_3 = 0.0, 5.5224\text{E-}001$, Freq$_3$ = 10.0 GHz).

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